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# ECONOMICAL ANALYSIS FOR INVESTMENT ON MEASURING SYSTEMS

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## ABSTRACT

Metrology processes contribute to entire manufacturing systems that can have a considerable impact on financial investment in coordinate measuring systems. However, there is a lack of generic methodologies to quantify their economical value in today's industry. To solve this problem, a mathematical model is proposed in this paper by statistical deductive reasoning. This is done through defining the relationships between Process Capability Index, measurement uncertainty and tolerance band. The correctness of the mathematical model is proved by a case study. Finally, several comments and suggestions on evaluating and maximizing the benefits of metrology investment are given.

## KEYWORDS

Metrology, Measurement, Economical Analysis, Process Capability Index, Statistical Process Control.

List of Abbreviations and Annotations

| Denotation | Meaning |
|---|---|
| $C_p$ | Process capability index. |
| Σ | Standard deviation. |
| USL | Upper specification limits. |
| LSL | Lower specification limits. |
| T | Tolerance band, T = USL – LSL. |
| Subscript 1 | True value of the manufacturing process. |
| Subscript 2 | Observed value determined by measurement. |
| Subscript m | Measuring system. |
| U/T | Uncertainty-Tolerance ratio. |

## 1. INTRODUCTION

High volume and large scale assemblies and fabrications with complex surfaces are increasingly engaged in the capital intensive industries, such as the aerospace, automotive and ship building (*Maropoulos et al. 2008*). The metrology has come to play an important role in the modern manufacturing systems. The metrology process, which employs the measuring and inspection activities, will liaise all the phases of product lifecycle from design, manufacturing, in-service to after-market maintenance (*Cai et al. 2008, Kunzmann et al., 2005*). Hence, the economical analysis on purchasing new measuring systems and metrology processes is expected to deliver high rates of return by the manufacturers, who are also normally the heavy investors (*Quatraro 2003, Renishaw Plc 2008, Swann 1994*). However, in the current industry, there is a lack of generic methodologies to evaluate the economical value that the metrology process brings to entire manufacturing systems (*Schmitt et al 2010, Schmitt et al. 2010*). There are a few comprehensive methodologies, which can properly quantify the economical benefits of the inspection processes delivering to complex modern production environments (*Kunzmann et a., 2005)*.

The aim of this paper is to quantitatively estimate the value that the metrology processes add to the manufacturing system. To achieve the aim, we firstly discuss whether or not the inspection process can bring economic benefits to the manufacturers and the investors by a brief literature review as background study in Section 2. Then a mathematical model is successfully established to define the relationships between Process Capability Index ($C_p$), measurement uncertainty (U) and tolerance band ($T$) in Section 3, followed by a case study to prove the correctness of the mathematical model (Section 0). Finally, in Section 4, the conclusion is given that the metrology process is economically beneficial to modern manufacturing systems.

## 2. BACKGROUND STUDY

It is now well-agreed that the product life cycle is built up by several linked phases, including design, manufacturing, in-service and after-market maintenance (Figure 1). The customers' requirements are captured in the design phase, and the specifications of the products are defined. Meanwhile the suppliers are called in and the supply chain is established. Then the CAD model of the product is sent to the manufacturing phase, where the machining, inspection and assembly activities are engaged leading to the physical products. The products are distributed and serviced in the market, and require maintenance and overhaul service at certain times during their service periods *(Zheng et al. 2008)*. During this loop, the metrology is the fundamental tool to gain information and knowledge in all phases of the product lifecycle establishing links between these separate phases (*Maropoulos et al. 2008, Schmitt et al 2010* ). Therefore, the metrology plays a vital role in the quality control process to guarantee the product conformance. Metrology increases the productivity of the engineering economics, particularly in robust engineering projects.

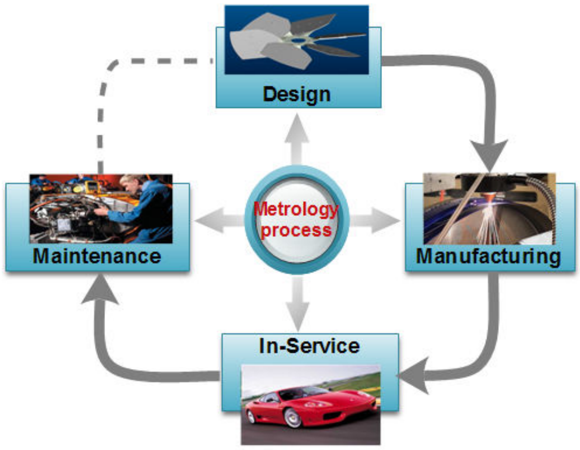


Figure 1. Roles of Metrology in Product Lifecycle

Historically concerns on metrology economics started in the 1970s, when computer aided measurement techniques were increasingly regarded as a means to control industrial manufacturing and quality of all kinds of products (*Osanna 2002*). In 1977, Peters raised the questions of 'why measure', 'what to measure' and 'how much the measurement pays' (*Peters 1977*). He was concerned with the macro-economic contribution of metrology in industrialized societies. But he did not specify the details of questions related to the micro-economical analysis. In 1993, Quinn, former director of Bureau International des Poids et Mesures (BIPM), stated that 'measurement and measurement related operations have been estimated to account for between 3% and 6% of the GDP of industrialized countries'. He concluded that the economics of metrology would increase further due to the continuous increase of high accuracy machining requirements and online measurement implementations (*Quinn, 1993*). Since then, investigations on the macro-economical impact of metrology have been conducted worldwide particularly across the UK (*Swann, 2009)*, EU (*L'Arrangement 2003*), US (*Tassey 1999, Tassey*

1999) and Japan (*McIntyre 1997*) by national authoritative organizations.

On the other hand, the micro-economics of metrology has been the subject of academic research and industrial applications for decades. Some researches focused on improving the efficiency and accuracy of CMM calibration procedures thus to save time and money. The temperature distribution along the machining surface was theoretically modelled and experimented to fast guide the groove depth in laser machining processes (*Chryssolouris and Yablon, 1993)*. The error sources of CMM measurement were identified and deliberately analyzed (*Savio, 2006*). This method provides a time-efficiency method to quick calibrate CMM performance. In aerospace and automotive industries, where large and complex parts are in demands, the data sampling techniques for inspection of free-form surfaces have been developed minimizing inspection costs and time (*Chryssolouris et al, 2001*).

Recently, some researches began to focus on identifying the costs and benefits of metrology as part of the entire industrial manufacturing system for robust engineering project. When considering the micro-economical value that processes add on the manufacturing systems, it is necessary to compare the costs of the processes against the benefits they generate (*Greeff, 2004*). The same applies to the metrology processes. It is relatively easy to calculate the cost of metrology using several established guidelines (*Semiconductor 2004, Semiconductor 2004*). However, the benefits analysis of the investment on the metrology equipment and processes remains problematic and challenging. This requires detailed considerations on fuzzy aspects, such as stability of production, measurement uncertainty, sample size and cost of risks (*Schmitt 2010*). And this is also what our research concentrates on.

To solve the above problem, a mathematical model is presented in this paper by preliminarily observing the calculating method to quantify the benefits of metrology process. An important parameter for statistical process control (SPC), the $C_p$, is introduced during the mathematical reasoning to reflect the cost effectiveness of metrology. Hence, in the next section (Section 3), a mathematical model related to $C_p$, $U$ and $T$ is established based on statistical analysis techniques and logical reasoning steps.

## 3. MATHEMATICAL MODEL

$C_p$ is a measurement parameter to indicate the ability of a process to produce outputs within specification limits resulting from the statistical analysis on the product quality of the entire manufacturing system (*NIST, 2010*). Since $C_p$ is derived from the mathematical statistics, in this section we firstly give a brief introduction on the normal distribution, which is frequently used in statistics and statistical process control to represent the mathematical model of the $C_p$. Then the sum of two normal distributions is discussed. Finally, the mathematical model is applied to the manufacturing environment, where the relationship between $C_p$, $U$ and nominal $T$ is defined.

### 3.1. TERMS RELATED TO NORMAL DISTRIBUTION

Normal distribution is one of the most frequently used mathematical models in probability theory and statistical analysis. It is a continuous probability distribution that describes, at least approximately, any variable that tends to cluster around the mean. As shown in Figure 2, the graph of normal distribution is bell shaped with a peak value at the mean (*Patel 1982*). In probability theory, the function which describes the relative likelihood of the occurrence of these continuous random variables at the mean value in the observation space is called probability density function (PDF).

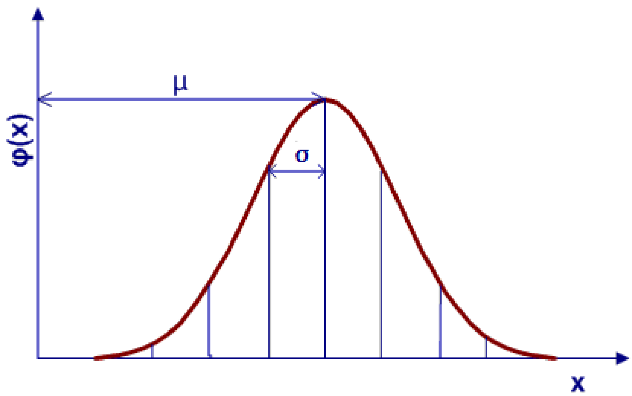


Figure 2. Normal distribution

The simplest case of a normal distribution is known as the standard normal distribution, of which PDF is described as

$$\varphi(x) = \frac{1}{2\pi} e^{-\frac{1}{2}x^2} \tag{1}$$

In more general cases, a normal distribution is derived from exponentiating a quadratic function:

$$f(x) = e^{ax^2+bx+c} \tag{2}$$

This provides the classical 'bell curve' shape of normal distribution. To describe the function expediently, rather than using $a$, $b$ and $c$, it is usually assumed that:

$$\mu = -\frac{b}{2a} \text{ and } \sigma^2 = -\frac{1}{2a}.$$

By changing the new parameters, the PDF is rewritten in a convenient standard form as

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} = \frac{1}{\sigma}\varphi\left(\frac{x-\mu}{\sigma}\right) \quad (3)$$

Equation (3) clearly shows that any normal distribution can be regarded as a version of the standard normal distribution that has been stretched horizontally by a factor $\sigma$ and then shifted rightward by a distance $\mu$ (as shown in Figure 2). In statistics, the parameter $\mu$ is called bias, which specifies the position of the bell curve's central peak. The parameter $\sigma^2$ is called the variance, which indicates how concentrated the distribution is close to its mean value. The square root of $\sigma^2$ is called the standard deviation and is the width of the density function.

Therefore, a normal distribution can be denoted as $N(\mu, \sigma^2)$. When a random variable X is distributed normally with mean $\mu$ and standard deviation $\sigma$, it is expressed as:

$$X \sim N(\mu, \sigma^2) \quad (4)$$

## 3.2. DETERMINING THE RELATIONSHIP BETWEEN $C_{P1}$, $C_{P2}$ AND (U/T)

A process is usually defined as a combination of tools, materials, methods, and people engaged in producing a measurable output, e.g. a production line for machined parts. Therefore all manufacturing processes are subjected to statistical variability which can be evaluated by statistical methods (*Altman, 2005*). To measure the variability of the manufacturing process, the process capability is defined and expressed in the form of $C_p$ to reflect how much 'natural variation' a process experiences relative to its specification limits. This allows for a comparison between different processes with respect to quality controls (*Greeff, 2004*).

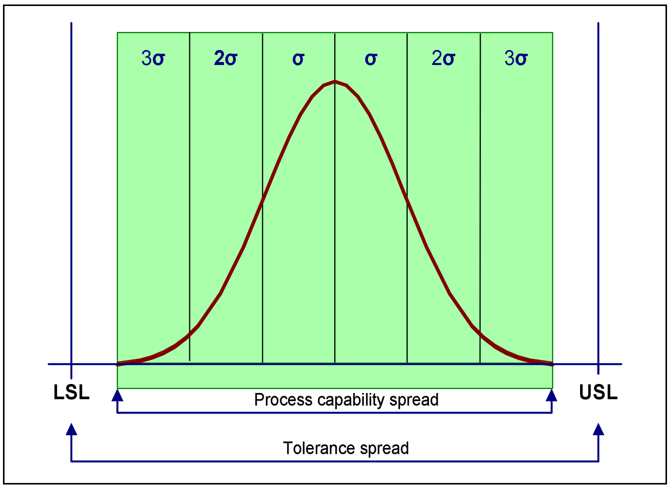


Figure 3. Process capability

The $C_p$ statistics is defined based on the assumption that measuring results of the manufactured parts are normally distributed (Figure 3). Assuming a two sided specification, if $\mu$ and $\sigma$ are the mean and standard deviation, respectively, of the normal data and *USL*, *LSL* are the upper and lower specification limits respectively, and then the process capability indices are defined as follows:

$$C_p = \frac{USL - LSL}{6\sigma}$$ for non-bias process, or

$$C_{pk} = \min\left(\frac{USL-\mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right)$$ for bias process (*Montgometry, 1996*).

In order to clarify the problem, this research has taken $C_p = \frac{USL - LSL}{6\sigma}$ (for non-bias process) for consideration and discussion. But the methodology developed from the research can be generalized to the bias manufacturing process with slight modifications (Please refer to 'Future Work').

As mentioned in previous section, the true value of $C_p$ that is observed after the measuring process can possibly be underestimated due to the intervention of the measurement uncertainty. For the convenience of understanding, we analogize the process capability reduction as the 'traffic lights 'to easily reveal 'go / no-go' decision making in real manufacturing environment (Figure 4). If a production manager receives a process capability evaluation result of a lower than expected $C_p$ value, he/she receives a 'red light' meaning he/she should stop the manufacturing, or purchase a more accurate machine tool or process. The unsatisfactory $C_p$ value may be caused by *U* from the inspection process (flashing as yellow light) instead of the machining process itself (green light). As the measurement instruments with lower uncertainties are generally cheaper than high precision machining tools, it is more economical to investment in proper measurement instruments or metrology processes. This provides the manufacturing system with a 'green light'.

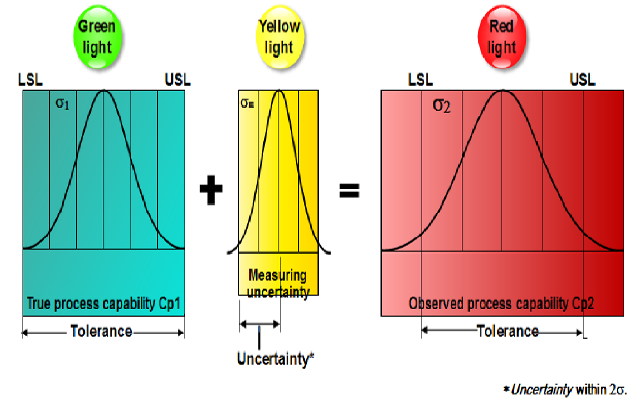


Figure 4. Influence of *U on* $C_p$.

So the next step of our work aims to quantitatively evaluate how much the observed $C_p$ value is reduced by the measurement uncertainty, and determine the true original value. The mathematical relationships between the true $C_p$ value, observed $C_p$ value and Uncertainty-Tolerance ratio are deduced by mathematical transformations.

It is assumed that the outputs of the manufacturing process are normally distributed, and well centred to the mean value, that is to say there is no bias ($\mu=0$).

From the definition of $C_p$, it follows that

$$C_{p1} = \frac{USL-LSL}{6\sigma_1} = \frac{T}{6\sigma_1}$$

and

$$C_{p2} = \frac{USL-LSL}{6\sigma_2} = \frac{T}{6\sigma_2}$$

thus

$$\sigma_1 = \frac{T}{6C_{p1}} \tag{5}$$

and

$$\sigma_2 = \frac{T}{6C_{p2}} \tag{6}$$

Assuming $U$ is within 2 σ , then

$$\sigma_m = \frac{U}{2} \tag{7}$$

Given the sum of normal distribution proved by Equation (A1) in Appendix A:

$$\sigma_2^{\ 2} = \sigma_1^{\ 2} + \sigma_m^{\ 2} \tag{8}$$

Inserting Equations (6), (7) and (8) into Equation (13),

$$(\frac{T}{6C_{p2}})^2 = (\frac{T}{6C_{p1}})^2 + (\frac{U}{2})^2 \tag{9}$$

Divided by T at both sides of the Equation (9) we achieve:

$$(\frac{1}{6C_{p2}})^2 = (\frac{1}{6C_{p1}})^2 + (\frac{1}{2}\times\frac{U}{T})^2 \tag{10}$$

The mathematical relationship between $C_{p1}$, $C_{p2}$ and U/T has been identified in Equation (10). The successful quantification of the reduction value of $C_p$ is due to measurement uncertainty. Note that U/T value is a popular parameter for the inspection engineers selecting measuring systems; therefore, Equation (15) can be utilized when making rational investment decisions on purchasing new machining or metrology systems.

## 4. METHOD VERIFICATION

In order to demonstrate the correctness of the final result (Equation (15)), a series of correlation curves were used to create explicably reflecting the correlation between $C_{p1}$, $C_{p2}$ and U/T by data acquisitioning and processing method.

**Table 1. Data collection to acquire the true $C_p$ value ($C_{p1}$)**

| U/T | $C_{p1}$=2.00 | $C_{p1}$=1.67 | $C_{p1}$=1.5 | $C_{p1}$=1.33 |
|---|---|---|---|---|
| 0 | 2 | 1.666667 | 1.5 | 1.333333 |
| 0.01 | 1.985754 | 1.658395 | 1.493962 | 1.329087 |
| 0.02 | 1.944775 | 1.634301 | 1.476275 | 1.316588 |
| 0.03 | 1.881775 | 1.596377 | 1.448144 | 1.296516 |
| 0.04 | 1.803046 | 1.547461 | 1.411331 | 1.269899 |
| 0.05 | 1.714986 | 1.490712 | 1.367882 | 1.237969 |
| 0.06 | 1.623069 | 1.429155 | 1.319858 | 1.202031 |
| 0.07 | 1.53141 | 1.365387 | 1.269137 | 1.163341 |
| 0.08 | 1.442775 | 1.301448 | 1.217302 | 1.123029 |
| 0.09 | 1.358816 | 1.238824 | 1.165596 | 1.082046 |
| 0.1 | 1.280369 | 1.178511 | 1.114941 | 1.041158 |
| 0.11 | 1.207715 | 1.121121 | 1.065977 | 1.000951 |
| 0.12 | 1.140792 | 1.066974 | 1.019112 | 0.96185 |
| 0.13 | 1.079332 | 1.016185 | 0.974581 | 0.924145 |
| 0.14 | 1.022968 | 0.96873 | 0.932487 | 0.888021 |
| 0.15 | 0.971286 | 0.9245 | 0.892841 | 0.853579 |
| 0.16 | 0.923869 | 0.883332 | 0.855594 | 0.820859 |
| 0.17 | 0.880314 | 0.845034 | 0.820653 | 0.789854 |
| 0.18 | 0.840247 | 0.809405 | 0.787904 | 0.760528 |
| 0.19 | 0.803323 | 0.776244 | 0.757219 | 0.732822 |
| 0.2 | 0.769231 | 0.745356 | 0.728464 | 0.706665 |
| 0.21 | 0.737691 | 0.716556 | 0.701509 | 0.681978 |
| 0.22 | 0.708454 | 0.689672 | 0.676225 | 0.658679 |
| 0.23 | 0.681298 | 0.664544 | 0.652489 | 0.636684 |
| 0.24 | 0.656023 | 0.641026 | 0.630185 | 0.615913 |
| 0.25 | 0.632456 | 0.618984 | 0.609208 | 0.596285 |
| 0.26 | 0.610437 | 0.598298 | 0.589456 | 0.577726 |
| 0.27 | 0.589829 | 0.578857 | 0.570838 | 0.560165 |
| 0.28 | 0.570507 | 0.560561 | 0.553268 | 0.543534 |
| 0.29 | 0.55236 | 0.543318 | 0.53667 | 0.527772 |
| 0.3 | 0.535288 | 0.527046 | 0.520972 | 0.512821 |
| 0.31 | 0.519202 | 0.511671 | 0.506107 | 0.498624 |
| 0.32 | 0.504023 | 0.497125 | 0.492018 | 0.485134 |
| 0.33 | 0.489679 | 0.483346 | 0.478647 | 0.472303 |
| 0.34 | 0.476104 | 0.470277 | 0.465946 | 0.460088 |
| 0.35 | 0.463241 | 0.457869 | 0.453869 | 0.448449 |
| 0.36 | 0.451037 | 0.446073 | 0.442372 | 0.437349 |
| 0.37 | 0.439443 | 0.434848 | 0.431418 | 0.426755 |
| 0.38 | 0.428416 | 0.424155 | 0.42097 | 0.416634 |
| 0.39 | 0.417916 | 0.413959 | 0.410996 | 0.406958 |
| 0.4 | 0.407909 | 0.404226 | 0.401466 | 0.3977 |

For data collection, it is assumed that there are five manufacturing systems, of which the true $C_p$ values ($C_{p1}$) are 2.00, 1.75, 1.50, 1.25, 1.00 and 0.75 respectively. Given the fact that the inspection engineers consider U/T as an important reference parameter for measurement instrument selection, Fthe U/T in Equation (15) is treated as a variable continuously varying between 0 and 1, which means that the measuring systems selected by the inspection engineers changed from the perfect measurement machines (without incurring any uncertainty) to the worst situation. Finally, the observed $C_p$ values ($C_{p2}$) for each case of the instrument selection (changing U/T values) under various manufacturing systems ($C_{p1}$) are acquired via Equation (10) . This reveals how the manufacturing system and its capability index have been influenced by the measurement uncertainty.

Following the method described above, a part of the data collection and acquisition is listed in Table 1. Then the curve is constructed as shown in Figure 5, which inexplicably reveals how the process capability drifts down due to the influence of the measurement instrument selection associated with the changes in the measurement uncertainties.

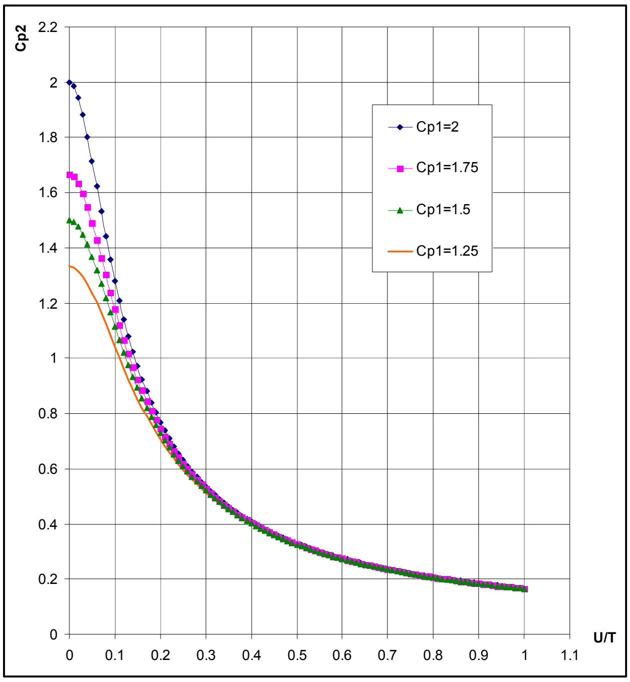


**Figure 5. Influence of U/T on $C_p$.**

## 5. DISCUSSION

### 5.1. CONSISTENT WITH ISO STANDARD AND NPL GOOD PRACTICE GUIDE

The result illustrated in Figure 5 has confirmed that the $C_p$ of a manufacturing system is underestimated due to the U introduced in the inspection and verification process, as discussed in Section 2. More importantly, the conclusion from this result is consistent with ISO 14253 (*ISO, 1998*) and NPL Good Practice Guide (*Flack and Hannaford, 2006*).

ISO 14253 provides clear guidelines about the need to allow for the uncertainty of the measurement instruments by reducing the size of the acceptance bands, and therefore argues that there is considerable interest in having access to measurement instruments with lower uncertainty (*ISO, 1998*). One step further, NPL Good Practice Guide No.80 (*Flack and Hannaford, 2006*) illustrates the impact of the *U* on the manufacturing process (Figure 6). It defines 'process tolerance' as an intangible tolerance variance that is developed from the nominal tolerance range, but is narrowed down as the *U* expands.

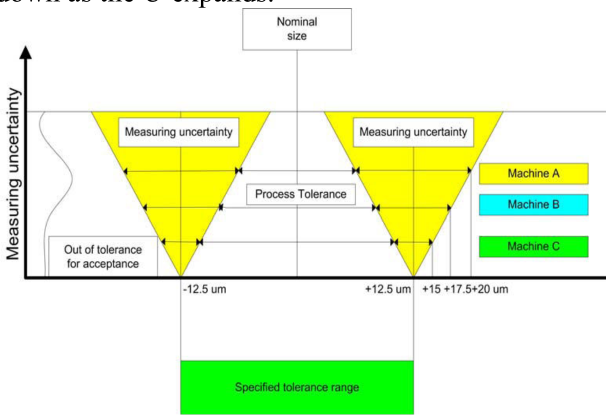


Figure 6. Impact of *U* (*Flack and Hannaford, 2006*).,

To this point we have further developed a mathematical method which quantitatively and directly discloses the impact of *U* on the process capability.

### 5.2. WHAT DO THE CURVES INDICATE?

The series of curves in Figure 5 put forward several suggestions on how U impacts on the manufacturing system.

Firstly, as discussed in Section 0, it can be seen that the observed $C_p$ value ($C_{p2}$) of the manufacturing system reduces as U increases. If we follow the common rule of metrology where U is considered to be 1/10 of the tolerance (U/T=10) for a finely-controlled process (*Cheng et al. 2009*), it can be read from Figure 5 that a 6σ quality controlled process of which the true $C_p$ value ($C_{p1}$) equals 2, will be underestimated at $C_p = 1.28$ due to the intervention of uncertainties from measurement and inspection processes. This result indicates that the process capability can be improved by purchasing new measuring systems. As normally a more accurate measurement instrument is cheaper than a more capable machining tool (*Kunzmann et al, 2005*), investing in new measuring systems and new inspection processes can be more economically viable than purchasing new machining tools.

Secondly, it can be seen that the higher the $C_p$ value that the manufacturing system has, the more sensitive the manufacturing systems will react towards the measurement instruments. This is consistent with the production engineers' common view that a highly capable machining system requires a highly accurate measuring system to inspect and verify the quality of the final products. It also suggests that generally the highly capable machining systems are worth the investment with a highly accurate measuring system.

Thirdly and the most importantly, our work has demonstrated that the economical value of the investment on the measurement equipment and the inspection processes can be quantitatively defined by the means of the measurement uncertainty. As the research continues, an accurate measurement instrument can possibly be evaluated by modelling the distribution density of the values of the parts' features and the effect of *U* on $C_p$ value, contributing to decision making for more complicated production environments. The investments on the metrology processes can generate direct value by reducing the scrap rate and production cost, thus it brings economical benefits for manufacturing systems.

## 6. CONCLUSION AND FUTURE WORK

The metrology process which involves measurement and inspection activities plays an increasingly vital role in the high value added manufacturing industries. The heavy investors, such as the manufacturers and venture capitalists, believe that proper investments in more capable metrology equipment and processes yield a high rate of return in capital intensive industries.

This paper reviews the state of the art metrology and dimensional measurement techniques within the current manufacturing industries. The question is how to evaluate the value added by metrology processes on the manufacturing systems. To answer the question, a mathematical model was successfully established by statistical deductive reasoning procedures, which defined the relationships between $C_p$, *U* and tolerance band, followed by a case study to verify the mathematical model. It is concluded that the metrology process is economically beneficial to modern manufacturing systems. Finally, several suggestions and comments on economical and productive investment in metrology systems were concluded based on the mathematical model derived and data collected during the case study.

In future work, an economical evaluation model for investing in measurement equipment will be developed based on the mathematical model in this paper. It will concern the question that how much it will cost if a manufacturer makes an error in the uncertainty zones around the tolerance limits. In other words, how much it costs to scrap a part that is actually in tolerance, and how much it costs if a non-conforming part is allowed to reach the customer. These decision making strategies will be developed by statistical analysis methodologies, and will deploy the risk management techniques which are commonly used in high value added capital investment.

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## APPENDIX A. SUM OF NORMAL DISTRIBUTIONS

In probability theory, the calculation of the sum of normal distributions is based on the distributions of the random variables involved and their relationships.

Conclusively speaking, the sum of the two independent normal distributions is also normally distributed, with the bias which is the sum of the two biases, and the variance which is the sum of the two variances (i.e., the square of the standard deviation is the sum of the squares of the standard deviations). The above can be expressed in mathematical formulae as:

if $X \sim N(\mu, \sigma^2)$ and $Y \sim N(\nu, \tau^2)$,

and they are independent, then

$$Z = X + Y \sim N(\mu + \nu, \sigma^2 + \tau^2) \qquad \text{(A1)}$$

The above proposition is proved using convolutional proof method (*Gohberg and Feldman, 2006*).

In mathematics analysis, convolution is a mathematical operation on two functions *f(x)* and *g(x)*, producing a third function that is typically viewed as a modified version of one of the original functions (*Gindikin, 1992*). In order to determine the sum of the normal distributions, according to the total probability theorem (*Mendenhall et al. 2005*), the probability density function of *z* is

$$f_Z(z) = \iint_{xy} f_{X,Y,Z}(x, y, z)dxdy$$

*X* and *Y* are independent, therefore

$$f_Z(z) = \iint_{xy} f_X(x) f_Y(y) f_Z(z \mid x, y)dxdy.$$

$f_Z(z \mid x, y)$ is trivially equal to $\delta(z-(x+y))$, where $\delta$ is Dirac's delta function (*Zill and Cullen, 2006*), therefore

$$f_Z(z) = \iint_{xy} f_X(x) f_Y(y) \delta(z-(x+y))dxdy. \qquad \text{(A2)}$$

as it was previously assumed that Z=X+Y, we substitute (z-x) for *y* in Equation (A2):

$$f_Z(z) = \int_x f_X(x) f_Y(z-x)dx \qquad \text{(A3)}$$

which is recognized as a convolution of $f_X$ with $f_Y$.

Therefore the PDF of the sum of the two independent random variables *X* and *Y* with PDFs *f* and *g* respectively is the convolution

$$(f * g)(x) = \int_{-\infty}^{+\infty} f(u) g(x-u)du$$

First, by assuming the two biases $\mu$ and $\nu$ are zero, the two PDFs are transformed as

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \quad \text{and} \quad g(x) = \frac{1}{\tau\sqrt{2\pi}} e^{-\frac{x^2}{2\tau^2}}.$$

The convolution becomes

$$(f\times g)(x)=\int_{-\infty}^{+\infty}(\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{u^2}{2\sigma^2}})\times(\frac{1}{\tau\sqrt{2\pi}}e^{-\frac{(x-u)^2}{2\tau^2}})du$$
$$=[cons]\int_{-\infty}^{+\infty}\exp(-\frac{u^2}{2\sigma^2})\times\exp(-\frac{(x-u)^2}{2\tau^2})du$$
$$=[cons]\int_{-\infty}^{+\infty}\exp(-\frac{u^2}{2\sigma^2}-\frac{(x-u)^2}{2\tau^2})du \qquad \text{(A4)}$$
$$=[cons]\int_{-\infty}^{+\infty}\exp(\frac{-(\tau^2u^2+\sigma^2(x-u)^2)}{2\sigma^2\tau^2})du$$

where *cons* is short for constant, and the below is the same.

Continuing the integral calculation from Equation (A4):

$$[cons]\int_{-\infty}^{+\infty}\exp(\frac{-(\tau^2u^2+\sigma^2(x-u)^2)}{2\sigma^2\tau^2})du \qquad \text{(A5)}$$
$$=[cons]\int_{-\infty}^{+\infty}\exp\left[\frac{-(\tau^2+\sigma^2)(u-\frac{\sigma^2}{\sigma^2+\tau^2}x)^2}{2\sigma^2\tau^2}+\frac{-x^2}{2(\sigma^2+\tau^2)}\right]du$$
$$=[cons]\times\exp\left[\frac{-x^2}{2(\sigma^2+\tau^2)}\right]\times\int_{-\infty}^{+\infty}\exp\left[\frac{-(\tau^2+\sigma^2)(u-\frac{\sigma^2}{\sigma^2+\tau^2}x)^2}{2\sigma^2\tau^2}\right]du$$

Note that the result of integral $\int_{-\infty}^{+\infty}\exp(-(u-A)^2)du$ does not depend on *A* (This can be proved by a simple substitution: $w = u - A$, $dw = du$, and the bounds of integration remain $-\infty$ and $+\infty$). Finally from the last part of Equation (A5), we obtain the convolution of PDFs *f(x)* and *g(x)* as below

$$(f\times g)(x)=[cons]\times\exp\left[\frac{-x^2}{2(\sigma^2+\tau^2)}\right]\times[cons] \qquad \text{(A6)}$$

where ‘constant’ means ‘not depending on variable *x*’.

Equation (A6) simply reveals that the sum of the two PDFs can be viewed as a constant multiple of $\exp\left[\frac{-x^2}{2(\sigma^2+\tau^2)}\right]$. In other words, the sum of normally distributed random variables is also normally distributed with the new variance of $(\sigma^2+\tau^2)$.

Therefore, the initial proposition in Equation (A1) has been proved. In Section 3.2, the equality relationship in Equation (A1) will be utilized to link the relationships between $C_p$, U and T.