Mechanics in deforming processes

Introduction

Deforming processes convert the original shape of a solid to another shape without changing its mass or material composition. During this process, cohesion is maintained among particles.

The relation between stress and strain for most solids is graphically described in Figure 1:

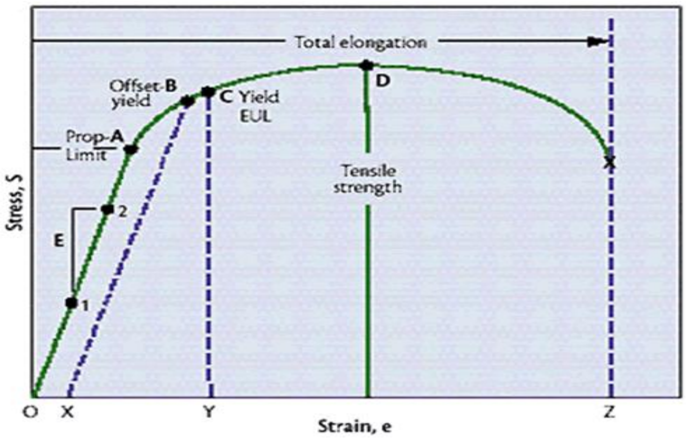

Figure 1: Stress-strain relationship.

1. For stress smaller than the point (A) of proportionality, stress and strain are proportional. Specifically $\sigma = E\varepsilon$, where E is Young's modulus. This is the elastic region
2. For stress larger than the proportionality limit, but smaller than the offset yield (point B), there is plastic deformation. In case of no more force is applied, the strain may follow a curve parallel to the elastic region, as indicated in Figure 1 by the dashed curve BX.
3. For stress larger than the offset yield, but smaller than the Extension-under-load (EUL) yield strength (point C), there is reduction of strain when force is no more applied, but not parallel to the elastic curve.
4. For stresses larger than the EUL yield strength, the whole strain generated by the applied stress remains as plastic deformation. Stress increases until it reaches the tensile strength. Further deformation does not correspond to further increase in the required stress, until it reaches the fracture point (the endpoint of Figure 1 curve).

Depending on the material and the region of the curve of Figure 1, several approximations to the curve of Figure 1 can be used. Such approximations are shown in Figure 2.

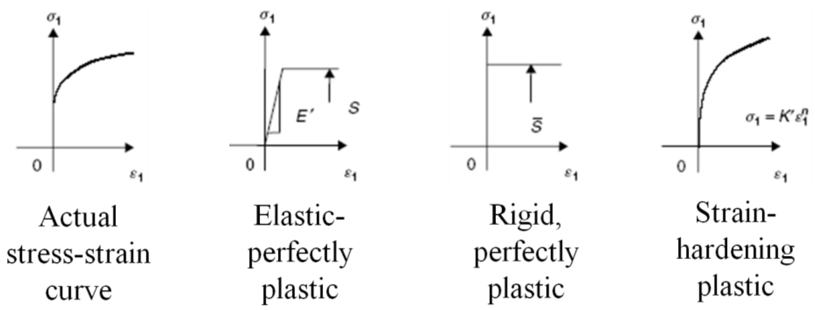

Figure 2: Models of stress-strain relationship.

The term stress (s) is used to express the loading in terms of force applied to a certain cross-sectional area of an object. From the perspective of loading, stress is the applied force or system of forces that tends to deform a body. From the perspective of what is happening within a material, stress is the internal distribution of forces within a body that balance and react to the loads applied to it. The stress distribution may or may not be uniform, depending on the nature of the loading condition. For example, a bar loaded in pure tension will essentially have a uniform tensile stress distribution. However, a bar loaded in bending will have a stress distribution that changes with distance perpendicular to the normal axis.

Some common measurements of stress are:
Psi = lbs/in^2 (pounds per square inch)
ksi or kpsi = kilopounds/in^2 (one thousand or 10^3 pounds per square inch)
Pa = N/m 2 (Pascals or Newtons per square meter)
kPa = Kilopascals (one thousand or 10^3 Newtons per square meter)
GPa = Gigapascals (one million or 10^6 Newtons per square meter)

Stresses in most 2-D or 3-D solids are complex and need to be defined methodically. The internal force acting on a small area of a plane can be resolved into three components: one normal to the plane and two parallel to the plane. The normal force component divided by the area gives the normal stress (s), and parallel force components divided by the area give the shear stress (t). These stresses are average stresses as the area is finite, but when the area approaches zero, the stresses become stresses at a point. Since stresses are defined in relation to the plane that passes through the point under consideration, and the number of such planes is infinite, there appear an infinite set of stresses at a point. It can be proven that the stresses on any plane can be computed from the stresses on three orthogonal planes passing through the point. As each plane has three stresses, the stress tensor has nine stress components, which completely describe the state of stress at a point.

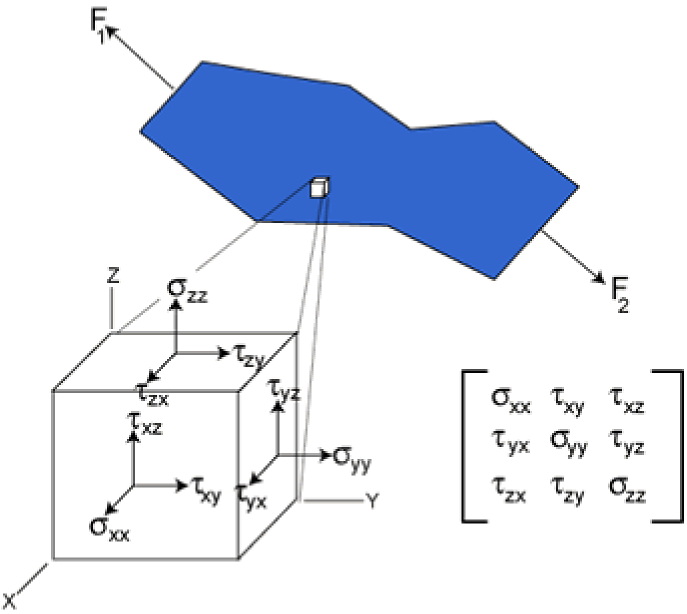

Figure 3a: Stresses in a Solid

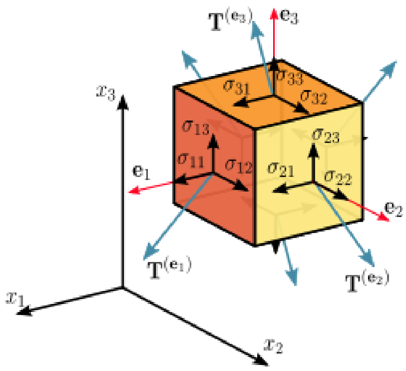

Figure 3b: Stresses on an infinitesimal part of a material.

As visible in Figure 3b, in order to completely describe the stresses, nine quantities have to be determined. They can be organized in the form of a matrix, like

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$

Due to physical reasons (conservation of angular momentum), the stress tensor is symmetric. Since the matrix is symmetric, its eigenvalues are real, they are called the principal stresses and notated as σ_1, σ_2 and σ_3, or σ_x, σ_y and σ_z.

Strain is the response of a system to an applied stress. When a material is loaded with a force, it produces a stress, which then causes a material to deform. Engineering strain is defined as the amount of deformation in the direction of the applied force divided by the initial length of the material. This results in a number without units, although it is often left in a form, such as inches per inch or meters per meter. For example, the strain in a bar that is being stretched in tension is the amount of elongation or change in length divided by its original length. As in the case of stress, the strain distribution may or may not be uniform in a complex structural element, depending on the nature of the loading condition

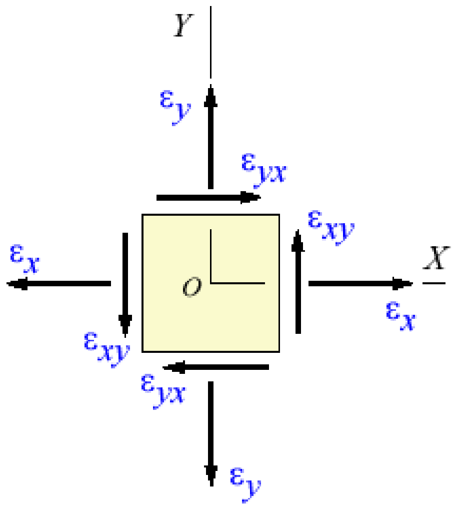

Figure 4: Strains on a 2D infinitesimal part of a material.

As shown in Figure 4, the strain, like the stress, can be organized in the form of a matrix. Figure 4 shows the strain for a two dimensional problem; in general, for a three dimensional problem the strain can be organized as

$$\varepsilon = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{pmatrix}$$

Ultimate tensile strength (UTS), often shortened to *tensile strength* (TS) or ultimate strength, is the maximum stress that a material can withstand while being stretched or pulled before failing or breaking. Tensile strength is not the same as compressive strength and the values can be quite different. It is the strength at point C of Figure 1 and is usually notated by σ_f or σ_y.

Yielding

Yielding is a particular point at the stress - strain curve, where the plastic behavior begins. This happens typically at 0.2% strain. There are several criteria for identifying the conditions of yielding:

<u>Maximum Principal Stress:</u>

Plastic deformation starts when the maximum of the principal stresses σ exceeds a given threshold: $\sigma \geq \sigma_y$.

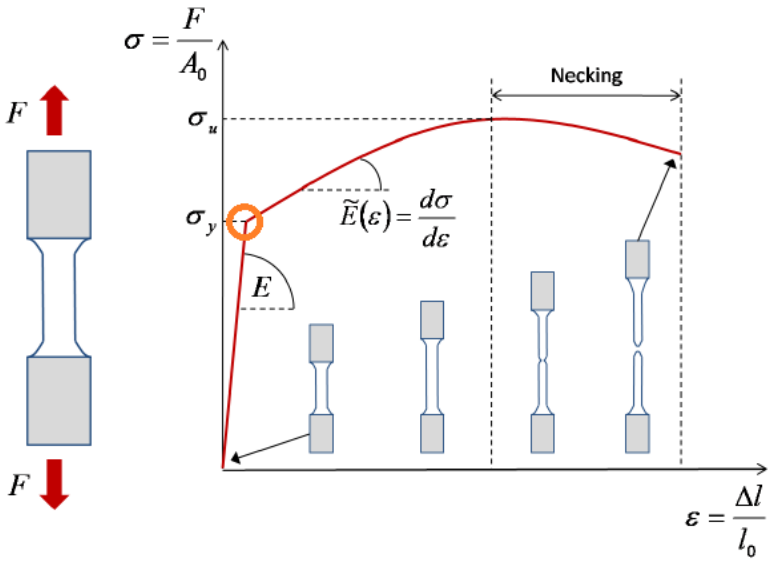

Figure 5: Maximum stress criterion (Yielding notated by circle).

Tresca Criterion:

A threshold is set for the shear stress (dislocation). Plastic deformation is considered to occur when $\sigma_1 - \sigma_3 \geq \sigma_y$.

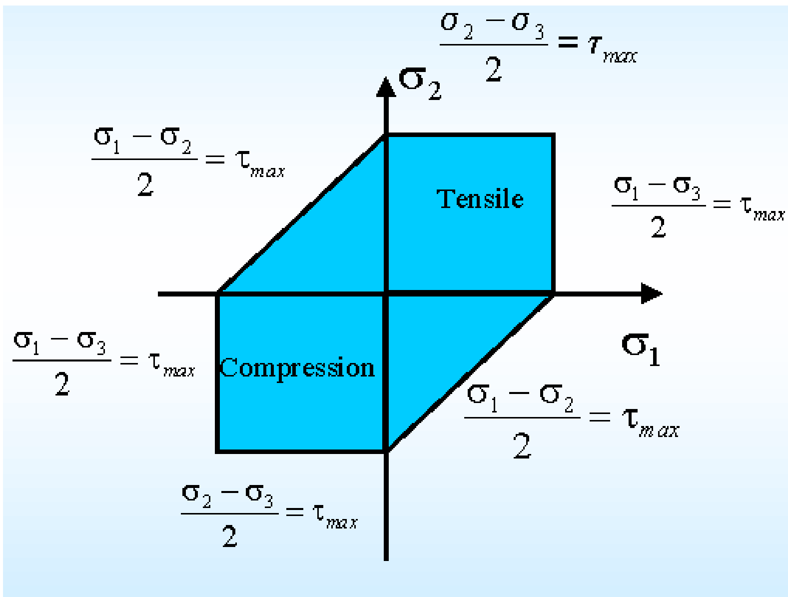

Figure 6: Tresca criterion for the case of Figure 3 ($\tau_{max} = \sigma_f/2$).

von Mises Criterion:

Plastic deformation is considered occurring when $\sigma_v > \sigma_y$,

where σ_v is the von Mises stress or equivalent tensile stress which is defined as

$$\sigma_v = \sqrt{\tfrac{1}{2}[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2)]}.$$

The von Mises criterion is an energy-related criterion, as the square of the stresses is involved.

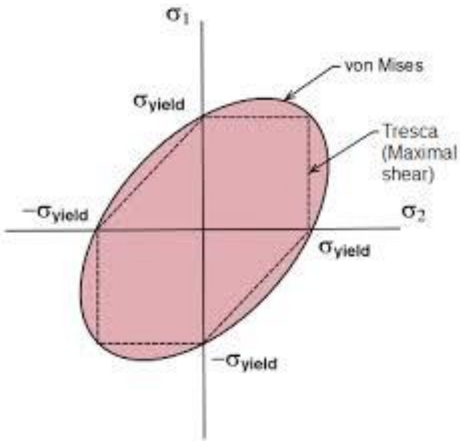

Figure 7: von Mises and Tresca criteria comparison for the case of Figure 3.

Bending Model

Figure 8 depicts the geometry of V-bending process in the x-y plane. A three point bending is taking place (two supports and one ram) and the result is bending the sheet forming to a desired angle. Technically, the depth (*z* dimension) is considered to be much larger than the other dimensions and every measure is expressed per unit of depth. This state is called "plane strain". Thus, every strain component involving *z* is equal to zero. Also, all the cross terms are set equal to zero, as the coordinate system (x,y) is supposed to be local at every point of the bended area.

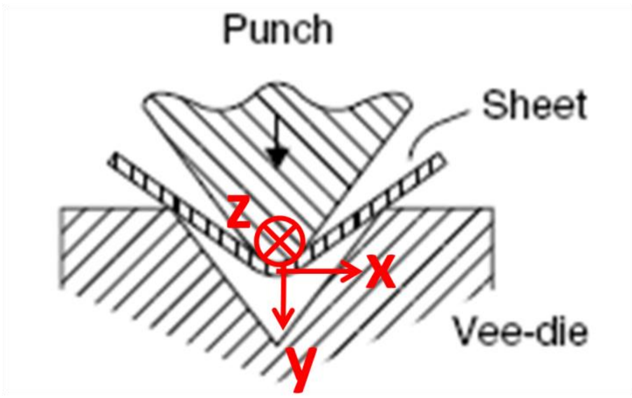

Figure 8: Bending Geometry.

One assumption is that **plane normal section remains plane and normal**.

Also, if l_o is the original length at the center, then line CD_o may change its length to CD and it is stretched to a new length l_s during bending (Figure 9):

$$l_s = \rho\theta$$

The line AB_0 (Figure 9) at the distance *y* from the center will deform to *AB*:

$$l = (\rho + y)\theta = l_s(1 + y/\rho)$$

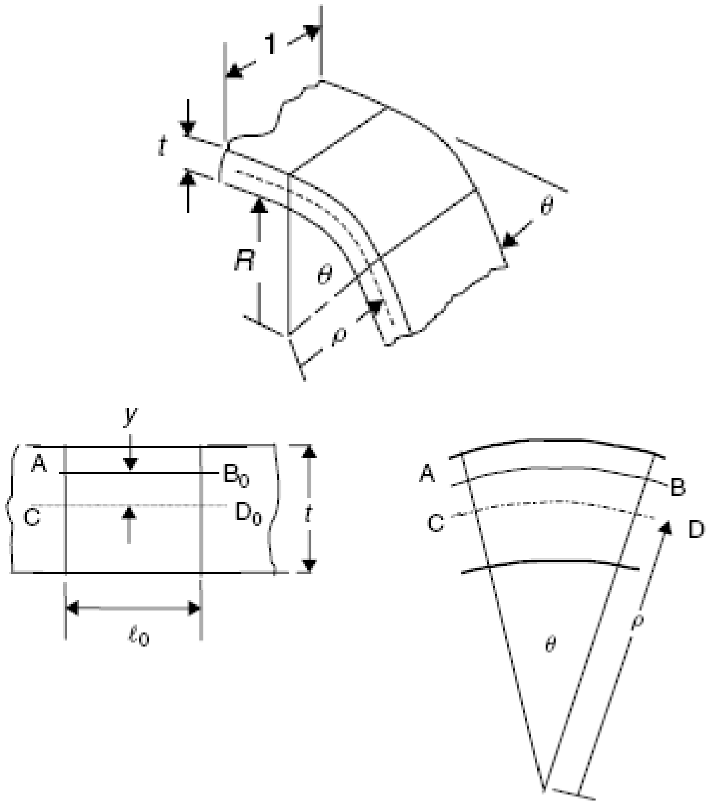

Figure 9: Bending Geometry Detail.

The axial strain at the line AB is

$$\varepsilon_1 = \int_{l_o}^{l} \frac{dl}{l} = \ln l - \ln l_o = \ln\frac{l}{l_o} = \ln\frac{l_s}{l_o} + \ln(1+\frac{y}{\rho}) = \varepsilon_a + \varepsilon_b$$

where:

- ε_α is the strain at the middle surface or the membrane strain
- ε_b is the so called bending strain

Finally, one can claim that where the radius of the curvature is large compared with the thickness, the bending strain can be simplified:

$$\varepsilon_b = \ln(1+\frac{y}{\rho}) \approx \frac{y}{\rho}$$

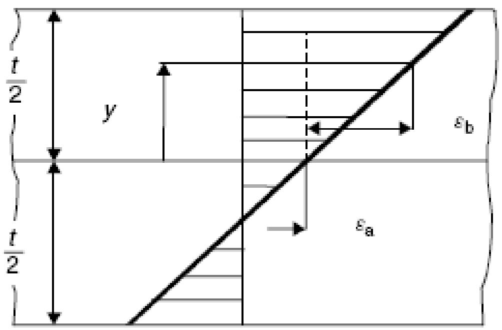

Figure 10: Linear approximation of the strain distribution at plane normal section.

Finally, there are considered to be three components of stress and strain. Concerning the strains, since an incompressible material is considered, the following relation (incompressibility condition) has to be taken into consideration:

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = 0$$

So, one can easily derive that:

$$\varepsilon_y = -\varepsilon_x$$

As far as the stresses are concerned, only pure bending moments are concerned, thus:

$$\sigma_y = 0$$

Regarding the other two components, the following formula has to be used:

$$\sigma_z = v_{pl}\sigma_x$$

As v_{pl}, one has to define a Poisson ratio for the case of incompressibility, thus a value of ½ is used.

Bending Numerical Example

In the following figure, the geometry is set. Also, in the following table, the data of the problem are given. What is asked is the Force P that achieves a bending of $\theta = 15°$.

Description	Variable	Value
Final bending angle	θ	$(180-15)°$
Thickness of metal sheet	t	1 mm
Material model (annealed 304 stainless steel, strain hardening model of Figure 2)	$\sigma = K\varepsilon^n$	K=430 MPa, n=0.45
Radius of Ram nose	R	2 mm
Friction Coefficient	μ	0.15
Distance between B_1 and B_2	$\lambda \triangleq \overline{B_1 B_2}$	8 mm

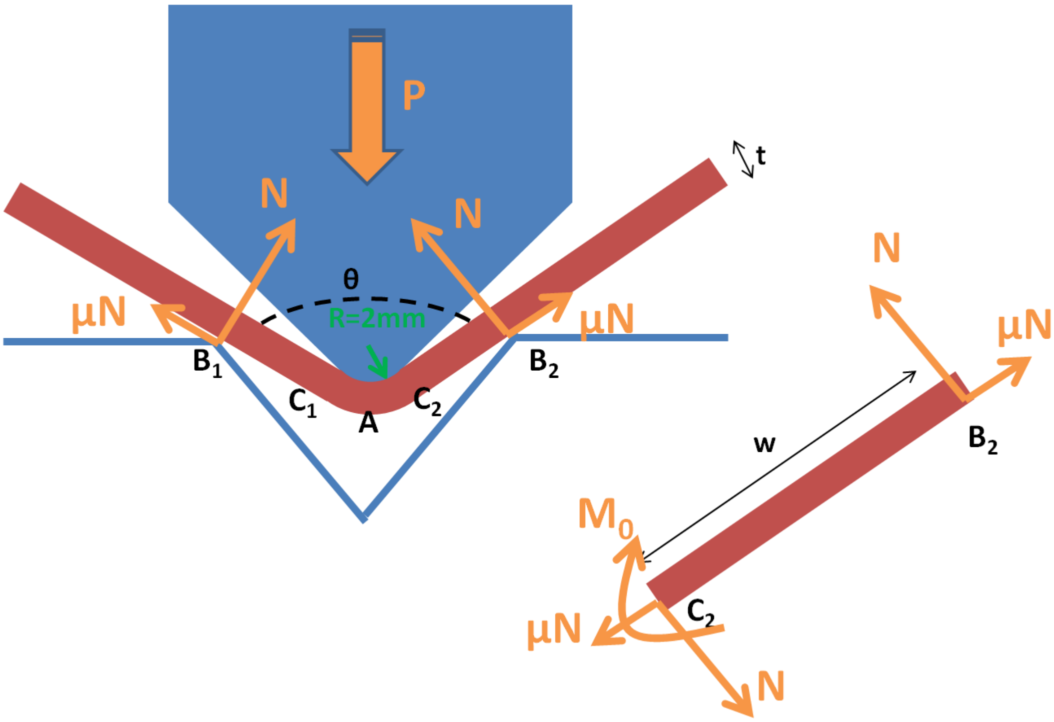

Figure 11: Bending geometry for numerical example

From the equilibrium of forces in the y direction (for the whole body), this is what is derived:

$$P = 2N\left(cos\frac{\theta}{2} + \mu sin\frac{\theta}{2}\right)$$

From the equilibrium of moments of the C_2B_2 part, the following formula is obtained:

$$N = \frac{2M_0}{2w + \mu t}$$

Regarding the length B_2C_2, taking into account Figure 12, it can be computed using:

$$(t+R)sin\varphi + wsin\frac{\theta}{2} = \frac{\lambda}{2}$$

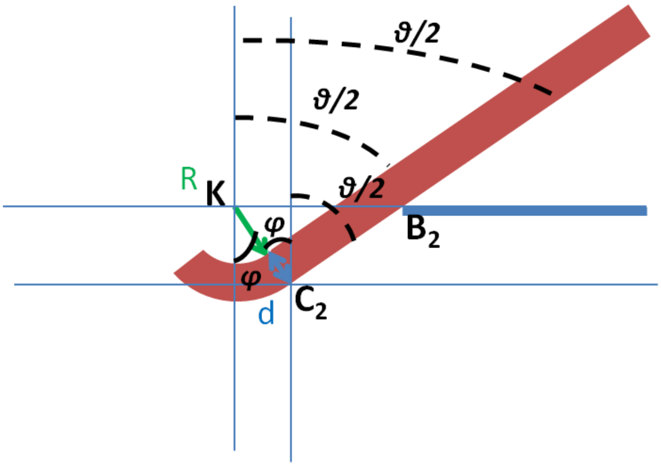

Figure 12: Detail in geometry

Then, using the fact that $\overrightarrow{KC_2} \perp \overrightarrow{B_2C_2} \Longrightarrow \varphi = \frac{\pi}{2} - \frac{\theta}{2}$,

$$(t+R)sin\varphi + wsin\frac{\theta}{2} = \frac{\lambda}{2}$$

$$w = \frac{\frac{\lambda}{2} - (t+R)sin\frac{\theta}{2}}{cos\frac{\theta}{2}} =..= 7.86mm$$

The bending moment is given by the following relationship:

$$M_0 = 2\int_0^{t/2} \sigma_x ydy = 2K\int_0^{t/2} \varepsilon_1^n ydy = 2K\int_0^{t/2} \left(\frac{y}{R'}\right)^n ydy = K\left(\frac{1}{R'}\right)^n \frac{t^{n+2}}{(n+2)2^{n+1}} =..$$
$$= 42.5Nm/m$$

In continuation,

$$N = 5.1\ kN/m$$

and finally:

$$\boxed{P \cong 9\ kN/m}$$

Assumptions that were taken into consideration:

- The area outside the supports B_1 & B_2 is not of any interest.
- The forces create only bending moment
- The outer curvature of the work-piece is driven by the ram
- The area outside bending remained rigid, whilst the bended area is plastic and is governed by power law stress-strain relationship.
- There is no spring-back effect
- Along the curved area, moment remains constant
- There are no thermal terms in modeling
- Every measure is calculated per unit of depth (plane strain)

Upsetting Model

Upsetting is a different deforming process, used to change the shape of a workpiece, as shown in Figure 13. Because of the geometry of the problem polar cylindrical coordinates will be used.

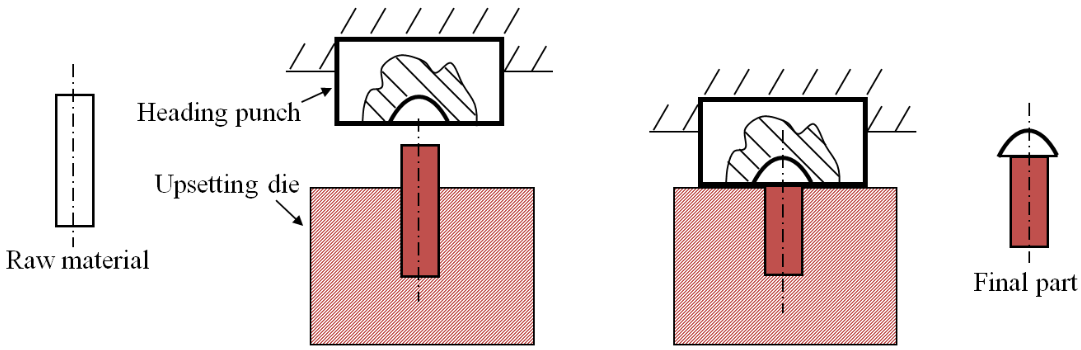

Figure 13: Upsetting

The determination of upsetting forces is given by the equilibrium in the radial direction, in the case of Figure 14.

$$\sigma_r r l d\varphi - (\sigma_r + d\sigma_r)(r + dr) l d\varphi + 2\sigma_t sin\frac{d\varphi}{2} l dr - 2\mu\sigma_z r d\varphi dr = 0$$

Assuming that $\sigma_r = \sigma_t$, that $sina \approx a$ and if all higher order derivatives second rank variations are set equal to zero, then one comes up with:

$$\frac{d\sigma_r}{dr} + \frac{2\mu}{l}\sigma_z = 0$$

Then, applying the Tresca yield criterion one finds:

$$\sigma_r - \sigma_z = \sigma_f$$

the following equation comes up:

$$\frac{d\sigma_r}{dr} + \frac{2\mu}{l}\sigma_r = \frac{2\mu}{l}\sigma_f$$

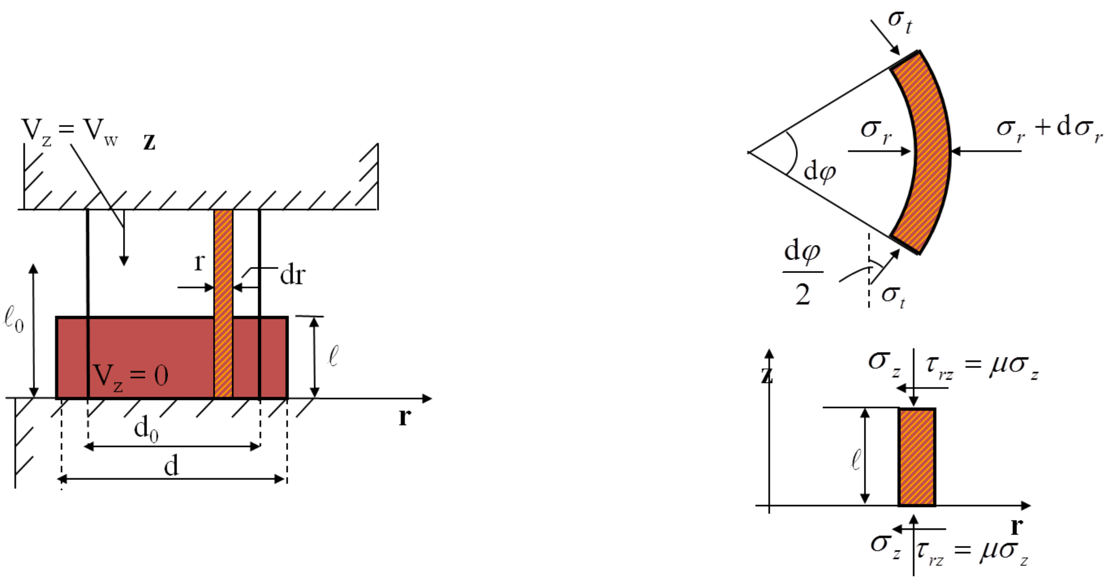

Figure 14: Modeling Upsetting

The solution to this differential equation using the boundary condition $\sigma_r|_{r=d/2} = 0$ is the following:

$$\sigma_r = -\sigma_f\left[e^{\frac{2\mu}{l}\left(\frac{d}{2}-r\right)} - 1\right], \sigma_z = -\sigma_f e^{\frac{2\mu}{l}\left(\frac{d}{2}-r\right)}$$

Using the Taylor expansion, the following formula can be valid:

$$\sigma_z = -\sigma_f\left[1 + \frac{2\mu}{l}\left(\frac{d}{2} - r\right)\right]$$

Also, it is worth mentioning that in the case of frictionless upsetting, $\frac{d\sigma_r}{dr} = 0$, thus $\sigma_z = -\sigma_f$.

Upsetting Numerical Example #1

Using the following values, a numerical example is formed.

Description	Variable	Value
Initial height	l_0	8 cm
Final height	l	6 cm
Initial radius	d_0	5 cm
Friction Coefficient	μ	0.5
Yield Strength for Steel	σ_f	300 MPa

The upsetting force is equal to:

$$F = \int_0^{d/2} \int_0^{2\pi} \sigma_z \rho d\varphi d\rho = -\sigma_f \int_0^{d/2} \int_0^{2\pi} \left(1 + \frac{2\mu}{l}\left(\frac{d}{2} - \rho\right)\right) \rho d\varphi d\rho$$

$$F = -2\pi\sigma_f \int_0^{d/2} \left(1 + \frac{2\mu}{l}\left(\frac{d}{2} - \rho\right)\right) \rho d\rho$$

$$F = -2\pi\sigma_f \left(\frac{d^2}{8} + \frac{d^3\mu}{24l}\right)$$

Making use of the fact that

$$\pi d_0^2 (l_0 - l)/4 = \pi l (d^2 - d_0^2)/4$$

the force is found to be equal to:

$$F = -2\pi\sigma_f \left(\frac{(d_0 - l + l_0)^2}{8} + \frac{(d_0 - l + l_0)^3 \mu}{24l}\right) =..= 900kN$$

Upsetting Numerical Example #2: Upsetting a nail

Using the following values, a numerical example is formed.

Description	**Variable**	**Value**
Initial height	l_0	0.5 mm
Final height	l	0.2 mm
Initial radius	d_0	1 mm
Friction Coefficient	μ	0.5
Yield Strength for Steel	σ_f	300 MPa

The upsetting force is found to be equal to:

$$F = 1300N$$